Chapter-3

Analysis of automated flow linear and line balancing

3.1 Need for Manual Assembly Lines:
Most manufactured consumer products are assembled. Each product consists of multiple components joined together by various assembly processes. These kinds of products are usually made on a manual assembly line. Factors favoring the use of manual assembly lines include the following:

- Demand for the product is high or medium.
- The products made on the line are identical or similar.
- The total work required to assemble the product can be divided into small work elements.
- It is technologically impossible or economically infeasible to automate the assembly operations.

Several reasons can be given to explain why manual assembly lines are so productive compared with alternative methods in which multiple workers each perform all of the tasks to assemble the products:

Specialization of labor: this principle asserts that when a large job is divided into small tasks and each task is assigned to one worker, the worker becomes highly proficient at performing the single task. Each worker becomes a specialist.

Interchangeable parts: here each component is manufactured to sufficiently close tolerances that any part of a certain type can be selected for assembly with its mating component. Without interchangeable parts, assembly would require filing and fitting of mating components, rendering assembly line methods impractical.

Work principle in material handling, which provides that each work unit flows smoothly through the production line, traveling minimum distances between stations.

Line pacing: Workers on an assembly line are usually required to complete their assigned tasks on each product unit within a certain cycle time, which paces the line to maintain a specified production rate. Pacing is generally implemented by means of a mechanized conveyor.

3.2 Fundamentals of Manual Assembly Lines
A manual assembly line is a production line that consists of a sequence of workstations where assembly tasks are performed by human workers, as depicted in Figure 3.1. Products are assembled as they move along the line. At each station, a portion of the total work is performed on each unit. The common practice is to "launch" base parts onto the beginning of the line at regular intervals. Each base part travels through successive stations and workers add components that progressively build the product. A mechanized material transport system is typically used to move the base part along the line as it is gradually transformed into the final product. However, in some manual lines, the product is simply moved manually from station-to-station. The production rate of an assembly line is determined by its slowest station. Stations capable of working faster are ultimately limited by the slowest station.

3.3 Assembly workstations:
A workstation on a manual assembly line is a designated location along the work flow path at which one or more work elements are performed by one or more workers. The work elements represent small portions of the total work that must be accomplished to assemble the product. A given workstation also includes the tools (hand tools or powered tools) required to perform the task assigned to the station.

Some workstations are designed for workers to stand, while others allow the workers to sit. When the workers stand, they can move about the station area to perform their assigned task. This is common for assembly of large products, such as cars, trucks, and major appliances. The typical case is when the product is moved by a conveyor at constant velocity through the station. The worker begins the assembly task near the upstream side of the station and moves along with the work unit until the task is completed, then walks...
back to the next work unit and repeats the cycle. For smaller assembled products (such as small appliances, electronic devices, and subassemblies used on larger products), the workstations are usually designed to allow the workers to sit while they perform their tasks. This is more comfortable and less fatiguing for the worker and is generally more conducive to precision and accuracy in the assembly task.

For a manual assembly line, the manning level of workstation $i$, symbolized $M_i$, is the number of workers assigned to that station; where $i = 1, 2, \ldots, n$; and $n =$ number of workstations on the line. The generic case is one worker: $M_i = 1$. In cases where the product is large, such as a car or a truck, multiple workers are often assigned to one station, so that $M_i > 1$. Multiple manning conserves valuable floor space in the factory and reduces line length and throughput time because fewer stations are required. The average manning level of a manual assembly line is simply the total number of workers on the line divided by the number of stations; that is,

$$M = \frac{w}{n} \quad \text{3.1}$$

Where $M =$ average manning level of the line (workers/station), $w =$ number of workers on the line, and $n =$ number of stations on the line. This seemingly simple ratio is complicated by the fact that manual assembly lines often include more workers than those assigned to stations, so that $M$ is not a simple average of $M_i$ values. These additional workers, called utility workers, are not assigned to specific workstations; instead they are responsible for functions such as (1) helping workers who fall behind, (2) relieving workers for personal breaks, and (3) maintenance and repair duties. Including the utility workers in the worker count, we have

$$M = \frac{w_u + \sum_{i=1}^{n} w_i}{n} \quad \text{3.2}$$

Where $w_u =$ number of utility workers assigned to the system; and $w_i =$ number of workers assigned specifically to station $i$ for $i = 1, 2, \ldots, n$. The parameter $w_i$ is almost always an integer, except for the unusual case where a worker is shared between two adjacent stations.

### 3.4 Work Transport Systems

There are two basic ways to accomplish the movement of work units along a manual assembly line: (1) manually or (2) by a mechanized system. Both methods provide the fixed routing (all work units proceed through the same sequence of stations) that is characteristic of production lines.

**Manual Methods of Work Transport:** In manual work transport, the units of product are passed from station-to-station by hand. Two problems result from this mode of operation are starving and blocking. Starving is the situation in which the assembly operator has completed the assigned task on the current work unit, but the next unit has not yet arrived at the station. The worker is thus starved for work. When a station is blocked, it means that, operator has completed the assigned task on the current work unit but cannot pass the unit to the downstream station because that worker is not yet ready to receive it. The operator is therefore blocked from working.

To mitigate the effects of these problems, storage buffers are sometimes used between stations. In some cases, the work units made at each station are collected in batches and then moved to the next station. In other cases, work units are moved individually along a flat table or unpowered conveyor. When the task is finished at each station, the worker simply pushes the unit toward the downstream station. Space is often allowed for one or move work units in front of each workstation. This provides an available supply of work for the station as well as room for completed units from the upstream station. Hence, starving and blocking are minimized. The trouble with this method of operation is that it can result in significant work-in-process, which is economically undesirable. Also, workers are unoccupied in lines that rely on manual transport methods, and production rates tend to be lower.

**Mechanized Work Transport:** Powered conveyors and other types of mechanized material handling equipment are widely used to move units along a manual assembly line. These systems can be designed to provide paced or unpaced operation of the line. Three major categories of work transport systems in production lines are: (a) continuous transport, (b) synchronous transport, and (c) asynchronous transport. These are illustrated schematically in Figure 3.2.

A continuous transport system uses a continuously moving conveyor that operates at constant velocity, as in Figure 3.2(a). This method is common on manual assembly lines. The conveyor usually runs the entire length of the line. However, if the line is very long, such as the case of an automobile final assembly plant, it is divided into segments with a separate conveyor for each segment. Examples of this kind are overhead trolley conveyor, Belt conveyor, Roller conveyor, Drag chain conveyor.

Continuous transport can be implemented in two ways: (1) Work units are fixed to the conveyor, and (2) work units are removable from the conveyor. In the first case, the product is large and heavy (e.g.,
automobile, washing machine) and cannot be removed from the conveyor. The worker must therefore walk along with the product at the speed of the conveyor to accomplish the assigned task.

In the case where work units are small and lightweight, they can be removed from the conveyor for the physical convenience of the operator at each station. Another convenience for the worker is that the assigned task at the station does not need to be completed within a fixed cycle time. Flexibility is allowed each worker to deal with technical problems that may be encountered with a particular work unit. However, on average, each worker must maintain a production rate equal to that of the rest of the line. Otherwise, the line will produce incomplete units, which occurs when parts that were supposed to be added at a station are not added because the worker runs out of time.

Figure 3.2 Velocity-distance diagram and physical layout for three types of mechanized transport systems used in production lines: (a) continuous transport, (b) synchronous transport, and (c) asynchronous transport. Key: \( v \) = velocity, \( v_c \) = constant velocity of continuous transport conveyor, \( x \) = distance in conveyor direction, \( \text{Sta} \) = workstation, \( i \) = workstation identifier.

In synchronous transport systems, all work units are moved simultaneously between stations with a quick, discontinuous motion, and then positioned at their respective stations. Depicted in Figure 3.2(b), this type of system is also known as intermittent transport, which describes the motion experienced by the work units. Synchronous transport is not common for manual lines, due to the requirement that the task must be completed within a certain time limit. This can result in incomplete units and excessive stress on the assembly workers. Despite its disadvantages for manual assembly lines, synchronous transport is often ideal for automated production lines. Examples of this kind are Walking beam transport equipment and Rotary indexing mechanisms.

In an asynchronous transport system, a work unit leaves a given station when the assigned task has been completed and the worker releases the unit. Work units move independently rather than synchronously. At any moment, some units are moving between workstations, while others are positioned at stations, as in Figure 17.2(c). With asynchronous transport systems, small queues of work units are permitted to form in front of each station. This tends to be forgiving of variations in worker task times. Examples of this kind are Power-and-free overhead conveyor, Carton track conveyor, Powered roller conveyors, automated guided vehicle system, Monorail systems, and Chain-driven carousel systems.

### 3.5 line pacing

A manual assembly line operates at a certain cycle time, which is established to achieve the required production rate of the line. On average, each worker must complete the assigned task at his/her station within this cycle time, or else the required production rate will not be achieved. This pacing of the workers is one of the reasons why a manual assembly line is successful. Pacing provides a discipline for the assembly line workers that more or less guarantees a certain production rate. From the viewpoint of management, this is desirable.

Manual assembly lines can be designed with three alternative levels of pacing:

- Rigid pacing,
- Pacing with margin, and
- No pacing.

In rigid pacing, each worker is allowed only a certain fixed time each cycle to complete the assigned task. The allowed time in rigid pacing is (usually) set equal to the cycle time of the line. Rigid pacing occurs when the line uses a synchronous work transport system. Rigid pacing has several undesirable aspects. First, in the performance of a repetitive task by a human worker, there is inherent variability in the time required to complete the task. This is incompatible with a rigid pacing discipline. Second, rigid pacing is emotionally and physically stressful to human workers. Although some level of stress is conducive to improved human performance, fast pacing on an assembly line throughout an 8-hr shift (or longer) can have harmful effects on workers. Third, in a rigidly paced operation, if the task has not been completed within the fixed cycle time, the work unit exits the station incomplete. This may inhibit completion of subsequent tasks at downstream stations. Whatever tasks are left undone on the work unit at the regular workstations must later be completed by some other worker to yield an acceptable product.
In pacing with margin, the worker is allowed to complete the task at the station within a specified time range. The maximum time of the range is longer than the cycle time, so that a worker is permitted to take more time if a problem occurs or if the task time required for a particular work unit is longer than the average. (This occurs when different product styles are produced on the same assembly line.) There are several ways in which pacing with margin can be achieved: (1) allowing queues of work units to form between stations, (2) designing the line so that the time a work unit spends inside each station is longer than the cycle time, and (3) allowing the worker to move beyond the boundaries of his/her own station. In method (1), work units are allowed to form queues in front of each station, thus guaranteeing that the workers are never starved for work, but also providing extra time for some work units as long as other units take less time. Method (2) applies to lines in which work units are fixed to a continuously moving conveyor and cannot be removed. Because the conveyor speed is constant, by designing the station length to be longer than the distance needed by the worker to complete the assigned task, the time spent by the work unit inside the station boundaries (called the tolerance time) is longer than the cycle time. In method (3), the worker is simply allowed to: (a) move upstream beyond the immediate station to get an early start on the next work unit or (b) move downstream past the current station boundary to finish the task on the current work unit. In either case, there are usually practical limits on how far the worker can move upstream or downstream, hence making this a case of pacing with margin. The terms upstream allowance and downstream allowance are sometimes used to designate these limits in movement. In all of these methods, as long as the worker maintains an average pace that matches the cycle time, the required cycle rate of the line will be achieved. The third level of pacing is when there is no pacing, meaning that no time limit exists within which the task at the station must be finished. In effect, each assembly operator works at his/her own pace. This case can occur when (1) manual work transport is used on the line, (2) work units can be removed from the conveyor, thus allowing the worker to take as much as time desired to complete a given unit, or (3) an asynchronous conveyor is used, and the worker controls the release of each work unit from the station. In each of these cases, there is no mechanical means of achieving a pacing discipline on the line. To reach the required production rate, the workers are motivated to achieve a certain pace either by their own collective work ethic or by an incentive system sponsored by the company.

3.6 Coping with Product Variety

Because of the versatility of human workers, manual assembly lines can be designed to deal with differences in assembled products. In general, the product variety must be relatively soft. Three types of assembly line can be distinguished:

- Single model,
- Batch model, and
- Mixed model

A single model line is one that produces many units of one product, and there is no variation in the Product. Every work unit is identical, and so the task performed at each station is the same for all product units. This line type is intended for products with high demand.

Batch model and mixed model lines are designed to produce two or more models, but different approaches are used to cope with the model variations. A batch model line produces each model in batches. Workstations are set up to produce the required quantity of the first model, then the stations are reconfigured to produce the next model, and so on. Products are often assembled in batches when demand for each product is medium. It is generally more economical to use one assembly line to produce several products in batches than to build a separate line for each different model.

When we state that the workstations are set up, we are referring to the assignment of tasks to each station on the line, including the special tools needed to perform the tasks, and the physical layout of the station. The models made on the line are usually similar, and the tasks to make them are therefore similar. However, differences exist among models so that a different sequence of tasks is usually required, and tools used at a given workstation for the last model might not be the same as those required for the next model. One model may take more total time than another, requiring the line to be operated at a slower pace. Worker retraining or new equipment may be needed to produce each new model. For these kinds of reasons, changes in the station setup are required before production of the next model can begin. These changeovers result in lost production time on a batch model line.

A mixed model line also produces more than one model; however, the models are not produced in batches. Instead, they are made simultaneously on the same line. While one model is being worked on at one station, a different model is being made at the next station. Each station is equipped to perform the variety of tasks needed to produce any model that moves through it. Many consumer products are assembled on mixed model lines. Examples are automobiles and major appliances, which are characterized by model variations, differences in available options, and even brand name differences in some cases.

Advantages of a mixed model line over a batch model line include:

- No lost production time switching between models,
- High inventories typical of batch production are avoided and
- Production rates of different models can be adjusted as product demand changes.
On the other hand, the problem of assigning tasks to workstations so that they all share an equal workload is more complex on a mixed model line. Scheduling (determining the sequence of models) and logistics (getting the right parts to each workstation for the model currently at that station) are more difficult in this type of line. And in general, a batch model line can accommodate wider variations in model configurations. As a summary of this discussion, Figure 3.3 indicates the position of each of the three assembly line types on a scale of product variety.

### 3.7 Alternative Assembly Systems

The well defined pace of a manual assembly line has merit from the viewpoint of maximizing production rate. However, assembly line workers often complain about the monotony of the repetitive tasks they must perform and the unrelenting pace they must maintain when a moving conveyor is used. Poor quality craftsmanship, sabotage of the line equipment, and other problems have occurred on high production assembly lines. To address these issues, alternative assembly systems are available in which either the work is made less monotonous and repetitious by enlarging the scope of the tasks performed, or the work is automated. In this section, following alternative assembly systems are identified:

- Single-station manual assembly cells,
- Assembly cells based on worker teams, and
- Automated assembly systems.

A **single-station manual assembly cell** consists of a single workplace in which the assembly work is accomplished on the product or some major subassembly of the product. This method is generally used on products that are complex and produced in small quantities, sometimes one-of-a-kind. The workplace may utilize one or more workers, depending on the size of the product and the required production rate. Custom-engineered products, such as machine tools, industrial equipment, and prototype models of complex products (e.g., aircraft, appliances, and cars) make use of a single manual station to perform the assembly work on the product.

**Assembly by worker teams** involves the use of multiple workers assigned to a common assembly task. The pace of the work is controlled largely by the workers themselves rather than by a pacing mechanism such as a powered conveyor moving at a constant speed. Team assembly can be implemented in several ways. A single station manual assembly cell in which there are multiple workers is a form of worker team. The assembly tasks performed by each worker are generally far less repetitious and broader in scope than the corresponding work on an assembly line.

Other ways of organizing assembly work by teams include moving the product through multiple workstations, but having the same worker team follow the product from station-to-station. This form of team assembly was pioneered by Volvo, the Swedish car maker. It uses independently operated automated guided vehicles that hold major components and/or subassemblies of the automobile and deliver them to manual assembly workstations along the line. At each station, the guided vehicle stops at the station and is not released to proceed until the assembly task at that station has been completed by the worker team. Thus, production rate is determined by the pace of the team, rather than by a moving conveyor. The reason for moving the work unit through multiple stations, rather than performing all the assembly at one station, is because the many component parts assembled to the car must be located at more than one station. As the car moves through each station, parts from that station are added. The difference between this and the conventional assembly line is that all work is done by one worker team moving with the car. Accordingly, the members of the team achieve a greater level of personal satisfaction at having accomplished a major portion of the car assembly. Workers on a conventional line who perform a very small portion of the total car assembly do not usually have this personal satisfaction.

The use of automated guided vehicles allows the assembly system to be configured with parallel paths, queues of parts between stations, and other features not typically found on a conventional assembly line. In addition, these team assembly systems can be designed to be high flexible, capable of dealing with variations in product and corresponding variations in product and corresponding variations in assembly cycle times at the different workstations. Accordingly, this type of team assembly is generally used when there are many different models to be produced, and the variations in the models result in significant differences in station service times.

Reported benefits of worker team assembly systems compared with conventional assembly line include:

- Greater worker satisfaction
- Better product quality
- Increased capability to accommodate model variations
greater ability to cope with problems that require more time rather than stopping the entire production line.

The principal disadvantage is that these team systems are not capable of the high production rates characteristic of a conventional assembly line.

Automated assembly systems use automated methods at workstations rather than humans. The term automated assembly refers to the use of mechanized and automated devices to perform the various assembly tasks in an assembly line or cell. Much progress has been made in the technology of assembly automation in recent years. Some of this progress has been motivated by advances in the field of robotics. Industrial robots are sometimes used as components in automated assembly systems.

3.8 Analysis of Single Model Assembly Lines

The relationships developed in this and the following sections are applicable to single model assembly lines. The assembly line must be designed to achieve a production rate $R_p$ sufficient to satisfy demand for the product. Product demand is often expressed as an annual quantity, which can be reduced to an hourly rate. Management must decide how many shifts per week the line will operate and how many hours per shift. Assuming the plant operates 50 wk/yr, then the required hourly production rate is given by

$$R_p = \frac{D_a}{50SH} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.3$$

Where $R_p =$ average production rate (units/hr), $D_a =$ annual demand for the single product to be made on the line (units/yr), $S =$ number of shifts/wk, and $H =$ number of hr/shift. If the line operates 52 wk rather than 50, then

$$R_p = \frac{D_a}{52SH} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.4$$

This production rate must be converted to a cycle time $T_c$ which is the time interval at which the line will be operated. The cycle time must take into account the reality that some production time will be lost due to occasional equipment failures, power outages, lack of a certain component needed in assembly, quality problems, labor problems, and other reasons. As a consequence of these losses, the line will be up and operating only a certain proportion of time out of the total shift time available; this uptime proportion is referred to as the line efficiency. The cycle time can be determined as

$$T_c = \frac{60E}{R_p} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.5$$

Where $T_c =$ cycle time of the line (min/cycle); $R_p =$ required production rate, (units/hr); the constant 60 converts the hourly production rate to a cycle time in minutes; and $E =$ line efficiency, the proportion of shift time that the line is up and operating. Typical values of $E$ for a manual assembly line are in the range 0.90 - 0.98. The cycle time $T_c$ establishes the ideal cycle rate for the line:

$$R_c = \frac{60}{T_c} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.6$$

Where $R_c =$ cycle rate for the line (cycles/hr), and $T_c$ is in min/cycle. This rate $R_c$ must be greater than the required production rate $R_p$ because the line efficiency $E$ is less than 100%. $R_p$ and $R_c$ are related to $E$ as follows:

$$E = \frac{R_p}{R_c} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.7$$

An assembled product requires a certain total amount of time to build, called the work content time $T_{wc}$. This is the total time of all work elements that must be performed on the line to make one unit of the product. It represents the total amount of work that is accomplished on the product by the assembly line. It is useful to compute a theoretical minimum number of workers that will be required on the assembly line to produce a product with known $T_{wc}$ and specified production rate $R_p$. The number of workstations required to achieve a specified production workload is given by

$$w = \frac{WL}{AT} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.8$$

Where $w =$ number of workers on the line; $WL =$ workload to be accomplished in a given time period; and $AT =$ available time in the period. The time period of interest will be 60 min. The workload in that period is the hourly production rate multiplied by the work content time of the product; that is,

$$WL = R_pT_{wc} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3.9$$

Where $R_p =$ production rate (pc/hr), and $T_{wc} =$ work content time (min/pc). Eq. (3.5) can be rearranged to the form $R_p = 60E/T_c$. Substituting this into Eq. (3.9), we have
The available time $AT = 1$ hr (60 min) multiplied by the proportion uptime on the line; that is, $AT = 60E$. Substituting these terms for $WL$ and $AT$ into Eq. (3.8), the equation reduces to the ratio $T_w/T_c$. Since the number of workers must be an integer, we can state:

$$w^* = \text{Minimum Integer} \geq \frac{T_w}{T_c} \quad \ldots \ldots 3.11$$

Where $w^*$ = theoretical minimum number of workers. If we assume one worker per station ($M_i = 1$ for all $i$, $i = 1, 2, \ldots, n$; and the number of utility workers $w_u = 0$), then this ratio also gives the theoretical minimum number of workstations on the line.

Achieving this minimum value in practice is very unlikely. Eq. (3.11) ignores several factors that exist in a real assembly line.

These factors tend to increase the number of workers above the theoretical minimum value:

**Repositioning losses**: Some time will be lost at each station for repositioning of the work unit or the worker. Thus, the time available per worker to perform assembly is less than $T_c$.

**The line balancing problem**: It is virtually impossible to divide the work content time evenly among all workstations. Some stations are bound to have an amount of work that requires less time than $T_c$. This tends to increase the number of workers.

**Task time variability**: There is inherent and unavoidable variability in the time required by a worker to perform a given assembly task. Extra time must be allowed for this variability.

**Quality problems**: Defective components and other quality problems cause delays and rework that add to the workload.

### 3.9 Repositioning losses

Repositioning losses on a production line occur because some time is required each cycle to reposition the worker or the work unit or both. For example, on a continuous transport line with work units attached to the conveyor and moving at a constant speed, time is required for the worker to walk from the unit just completed to the upstream unit entering the station. In other conveyorized systems, time is required to remove the work unit from the conveyor and position it at the station for the worker to perform his or her task on it. In all manual assembly lines, there is some lost time for repositioning. Let us define $T_r$, as the time required each cycle to reposition the worker or the work unit or both. In our subsequent analysis, we assume that $T_r$ is the same for all workers, although repositioning times may actually vary among stations.

![Figure 17.4 Components of cycle time at several stations on a manual assembly line. At the slowest station, the bottleneck station, idle time = zero; at other stations idle time exists. Key: Sta = workstation, $n$ = number of workstations on the line, $T_r$ = repositioning time, $T_s$ = service time, $T_c$ = cycle time.](image)

The repositioning time $T_r$ must be subtracted from the cycle time $T_c$, to obtain the available time remaining to perform the actual assembly task at each workstation. Let us refer to the time to perform the assigned task at each station as the **service time**. It is symbolized $T_s$, where $i$ is used to identify station $i$, $i = 1, 2, \ldots, n$. Service times will vary among stations because the total work content cannot be allocated evenly among stations. Some stations will have more work than others will. There will be at least one station at which $T_s$ is maximum. This is sometimes referred to as the **bottleneck station** because it establishes the cycle time for the entire line. This maximum service time must be no greater than the difference between the cycle time $T_c$ and the repositioning time $T_r$, that is,

$$\text{Max} \{T_s\} \leq T_c - T_r \quad \text{for } i = 1, 2, \ldots, n \quad \ldots \ldots 3.12$$

Where Max $\{T_s\}$ = maximum service time among all stations (min/cycle), $T_c$ = cycle time for the assembly line (min/cycle), and $T_r$ = repositioning time (assumed the same for all stations) (min/cycle). For simplicity of notation, let us use $T_s$ to denote this maximum allowable service time; that is,
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\[ T_s = \text{Max} \{ T_{ai} \} \leq T_c - T_i \]  \hspace{1cm} \text{(3.13)}

At all stations where \( T_{ai} \) is less than \( T_c \), workers will be idle for a portion of the cycle, as shown in Figure 3.4. When the maximum service time does not consume the entire lines available time \( T_c - T_i \), then this means that the line could be operated at a faster pace than \( T_c \), from Eq. (3.4). In this case, the cycle time \( T_c \) is usually reduced so that \( T_c = T_s + T_i \); this allows the production rate to be increased slightly.

Repositioning losses reduce the amount of time that can be devoted to productive assembly work on the line. These losses can be expressed in terms of an \textit{efficiency factor} as follows:

\[ E_r = \frac{T_s}{T_c} = \frac{T_c - T_i}{T_c} \]  \hspace{1cm} \text{(3.14)}

Where \( E_r \) = repositioning efficiency, and the other terms are defined above.

### 3.10 The Line Balancing Problem

The work content performed on an assembly line consists of many separate and distinct work elements. Invariably, the sequence in which these elements can be performed is restricted, at least to some extent. And the line must operate at a specified production rate, which reduces to a required cycle time as defined by Eq. (3.5). Given these conditions, the line balancing problem is concerned with assigning the individual work elements to workstations so that all workers have an equal amount of work.

Two important concepts in line balancing are the \textit{separation of the total work content into minimum rational work elements} and the \textit{precedence constraints} that must be satisfied by these elements. Based on these concepts we can define performance measures for solutions to the line balancing problem.

#### 3.10.1 Minimum Rational Work Elements:

A minimum rational work element is a small amount of work having a specific limited objective, such as adding a component to the base part or joining two components or performing some other small portion of the total work content. A minimum rational work element cannot be subdivided any further without loss of practicality. For example, drilling a through-hole in a piece of sheet metal or fastening two machined components together with a bolt and screw would be defined as minimum rational work elements. It makes no sense to divide these tasks into smaller elements of work. The sum of the work element times is equal to the work content time; that is,

\[ T_{wc} = \sum_{k=1}^{n_e} T_{ek} \]  \hspace{1cm} \text{(3.15)}

Where \( T_{ek} = \) time to perform work element \( k \) (min), and \( n_e = \) number of work elements into which the work content is divided; that is, \( k = 1, 2, ..., n_e \).

In line balancing, we make the following assumptions about work element times:
- Element times are constant values, and
- TA values are additive; that is; the time to perform two or more work elements in sequence is the sum of the individual element times.

In fact, we know these assumptions are not quite true. Work element times are variable, leading to the problem of task time variability. And there is often motion economy that can be achieved by combining two or more work elements, thus violating the additivity assumption. Nevertheless, these assumptions are made to allow solution of the line balancing problem.

The task time at station \( i \), or service time as we are calling it, \( T_{si} \), is composed of the work element times that have been assigned to that station; that is,

\[ T_{si} = \sum_{k \in i} T_{ek} \]  \hspace{1cm} \text{(3.16)}

An underlying assumption in this equation is that all \( T_{ek} \) are less than the maximum service time \( T_c \).

Different work elements require different times, and when the elements are grouped into logical tasks and assigned to workers, the station service times \( T_{si} \) are not likely to be equal. Thus, simply because of the variation among work element times, some workers will be assigned more work, while others will be assigned less. Although service times vary from station-to-station, they must add up to the work content time:

\[ T_{wc} = \sum_{i=1}^{n} T_{si} \]  \hspace{1cm} \text{(3.17)}

#### 3.10.2 Precedence Constraints:

In addition to the variation in element times that make it difficult to obtain equal service times for all stations, there are restrictions on the order in which the work elements can be performed.
Some elements must be done before others. For example, to create a threaded hole, the hole must be drilled before it can be tapped. A machine screw that will use the tapped hole to attach a mating component cannot be fastened before the hole has been drilled and tapped.

These technological requirements on the work sequence are called precedence constraints. They complicate the line balancing problem.

Precedence constraints can be presented graphically in the form of a precedence diagram, which indicates the sequence in which the work elements must be performed. Work elements are symbolized by nodes, and the precedence requirements are indicated by arrows connecting the nodes. The sequence proceeds from left to right. Figure 3.5 shows the precedence diagram.

### 3.11 Measures of Line Balance Efficiency

Because of the differences in minimum rational work element times and the precedence constraints among the elements, it is virtually impossible to obtain a perfect line balance. Measures must be defined to indicate how good a given line balancing solution is. One possible measure is balance efficiency, which is the work content time divided by the total available service time on the line:

\[
E_b = \frac{T_{wc}}{wT_s} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldOTS
Solution:
Given:
- $D_a = 100,000$ units/yr
- Duration: 50 wk/yr
- $S = 5$ shifts/wk
- $H = 7.5$ Hr/shift
- $E = 0.96$
- $T_r = 0.08$ min
- $T_{wc} = ?$
- $R_p = ?$
- $T_c = ?$
- $w^* = ?$
- $T_s = ?$

a) The total work content time is the sum of the work element times in the above Table. $T_{wc} = 4.0$ min
b) Given the annual demand, the hourly production rate is

$$R_p = \frac{D_a}{50SH} = \frac{100,000}{50 \times 5 \times 7.5} = 53.33 \text{ units/Hr}$$

c) The corresponding cycle time $T_c$, with an uptime efficiency of 96% is

$$T_c = \frac{60E}{R_p} = \frac{60 \times 0.96}{53.33} = 1.08 \text{ min}$$

d) The theoretical minimum number of workers is given by

$$w^* = \frac{T_{wc}}{T_c} = \frac{4.0}{1.08} = 3.70$$

As number of workers should be an integer, $w = 4$ workers

e) The available service time against which the line must be balanced is

$$T_s = T_c - T_r = 1.08 - 0.08 = 1.00 \text{ min}.$$ 

Example 2:

A manual assembly line must be designed for a product with annual demand = 100,000 units. The line will operate 50 wk/yr, 5 shifts/wk, and 7.5 hr/shift. Work units will be attached to a continuously moving conveyor. Work content time = 42.0 min. Assume line efficiency $E=0.97$, balancing efficiency $E = 0.92$ and repositioning time $T_r = 6$ sec. Determine: (a) hourly production rate to meet demand and (b) number of workers required.

Given:
- $D_a = 100,000$ units/yr
- Duration: 50 weeks
- $S = 5$ shifts/wk
- $H = 7.5$ Hr/shift
- $T_{wc} = 42.0$ min
- $E = 0.97$
- $E_b = 0.92$
- $T_r = 6$ sec
- $R_p = ?$
- $w = ?$

a) the hourly production rate is given by

$$R_p = \frac{D_a}{50SH} = \frac{100,000}{50 \times 5 \times 7.5} = 53.33 \text{ units/Hr}$$

b) we know that

$$E_b = \frac{T_{wc}}{wT_s}$$

where $w$ is to be determined, we don’t know $T_s$

But, $T_s = T_c - T_r$

Where $T_c = \frac{60E}{R_p} = \frac{60 \times 0.97}{53.33} = 1.091$ min

$$T_s = 1.091 - 6/60 = 0.991 \text{ min}$$

$$E_b = \frac{T_{wc}}{wT_s}; W = \frac{T_{wc}}{E_bT_s} = \frac{42.0}{0.92 \times 0.991} = 46.06$$

As number of workers should be an integer, $w = 46$ workers

Example 3:

A single model assembly line is being planned to produce a consumer appliance at the rate of 200,000 units/yr. The line will be operated 8 hr/shift, 2 shifts/day, 5 day/wk, 50 wk/yr. Work content time = 35.0 min. For planning purposes, it is anticipated that the proportion uptime on the line will be 95%. Determine: (a) average hourly production rate Rp (b) cycle time $T_c$ and (c) theoretical minimum number of workers required on the line. (d) If the balance efficiency is 0.93 and the repositioning time = 6 sec, how many workers will be required?

Given
- $D_a = 200,000$ units/yr
- Duration: 50 weeks
- $S = 2 \times 5$ shifts/wk
- $H = 8$ Hr/shifts
- $T_{wc} = 35.0$ min
- $E = 0.95$
- $E_b = 0.93$
- $T_r = 6$ sec
- $R_p = ?$
- $w = ?$

a) the hourly production rate is given by

$$R_p = \frac{D_a}{50SH} = \frac{200,000}{50 \times 10 \times 8} = 50 \text{ units/Hr}$$

b) we know that

$$T_c = \frac{60E}{R_p} = \frac{60 \times 0.95}{50} = 1.14 \text{ min}$$

c) The theoretical minimum number of workers is given by

$$w^* = \frac{T_{wc}}{T_c} = \frac{35.0}{1.14} = 30.70$$

As number of workers should be an integer, $w = 31$ workers
d) Actual number of workers required is given by

\[ E_w = \frac{T_{wc}}{wT_s} \]

Where \( w \) is to be determined, we don’t know \( T_s \)

But, \( T_s = T_c - T_r \)

Where \( T_s = 1.14 - 6/60 = 1.04 \) min

\[ E_w = \frac{T_{wc}}{wT_s} ; \quad W = \frac{T_{wc}}{E_wT_s} = \frac{35.0}{0.92 \times 0.991} = 36.58 \]

As number of workers should be an integer, \( w = 37 \) workers

### 3.12 Line Balancing Algorithms

The objective in line balancing is to distribute the total workload on the assembly line as evenly as possible among the workers. This objective can be expressed mathematically in two alternative but equivalent forms:

Minimize \((wT_s - T_{wc})\) or Minimize \( \sum_{i=1}^{w} (T_s - T_{sl}) \)

Subject to: (1) \( \sum_{k=1}^{i} T_{ek} \leq T_s \) and (2) all precedence requirements are obeyed.

The three methods of line balancing algorithms are: (1) largest candidate rule, (2) Kilbridge and Wester method, and (3) ranked positional weights method. These methods are heuristic, meaning they are based on common sense and experimentation rather than on mathematical optimization. In each of the algorithms, we assume that the manning level is one, so when we identify station \( i \), we are also identifying the worker at station \( i \).

#### 3.12.1 Largest candidate rule

This rule has the following four steps.

1) Work elements are arranged in descending order according to their \( T_{ek} \) values

2) Assign elements to the worker at the first workstation by starting at the top of the list and selecting the first element that satisfies precedence requirements and does not cause the total sum of \( T_{ek} \) at that station to exceed the allowable \( T_s \); when an element is selected for assignment to the station, start back at the top of the list for subsequent assignments

3) when no more elements can be assigned without exceeding \( T_s \) then proceed to the next station

4) Repeat steps 2 and 3 for the other stations in turn until all elements have been assigned.

#### Example: 4

Apply the largest candidate rule for the problem given.

A small electrical appliance is to be produced on a single model assembly line. The work content of assembling the product has been reduced to the work elements listed in Table below. The table also lists the standard times that have been established for each element as well as the precedence order in which they must be performed. The line is to be balanced for an annual demand of 100,000 unit/yr. The line will operate 50 wk/yr, 5 shifts/wk, and 7.5 hr/shift. Manning level will be one worker per station. Previous experience suggests that the uptime efficiency for the line will be 96%, and repositioning time lost per cycle will be 0.08 min. Determine: (a) total work content time \( T_{wc} \) (b) required hourly production rate \( R_p \) to achieve the annual demand, (c) cycle time \( T_c \) (d) Theoretical minimum number of workers required on the line, and (e) service time \( T_s \), to which the line must be balanced.

<table>
<thead>
<tr>
<th>No.</th>
<th>Work Element Description</th>
<th>( T_{ek} ) (min)</th>
<th>Must Be Preceded By</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Place frame in work holder and clamp</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Assemble plug, grommet to power cord</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Assemble brackets to frame</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Wire power cord to motor</td>
<td>0.1</td>
<td>1, 2</td>
</tr>
<tr>
<td>5</td>
<td>Wire power cord to switch</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>Assemble mechanism plate to bracket</td>
<td>0.11</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>Assemble blade to bracket</td>
<td>0.32</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>Assemble motor to brackets</td>
<td>0.6</td>
<td>3, 4</td>
</tr>
<tr>
<td>9</td>
<td>Align blade and attach to motor</td>
<td>0.27</td>
<td>6, 7, 8</td>
</tr>
<tr>
<td>10</td>
<td>Assemble switch to motor bracket</td>
<td>0.38</td>
<td>5, 8</td>
</tr>
<tr>
<td>11</td>
<td>Attach cover, inspect, and test</td>
<td>0.5</td>
<td>9, 10</td>
</tr>
<tr>
<td>12</td>
<td>Place in tote pan for packing</td>
<td>0.12</td>
<td>11</td>
</tr>
</tbody>
</table>
Solution:
Given:
- $D_a = 100,000$ unit/yr
- Duration: $50$ wk/yr
- $S = 5$ shifts/wk
- $H = 7.5$ Hr/shift
- $E = 0.96$
- $T_r = 0.08$ min
- $T_{wc} = ?$
- $R_p = ?$
- $T_c = ?$
- $w^* = ?$
- $T_s = ?$

a) The total work content time is the sum of the work element times in the above Table.
$$T_{wc} = 4.0 \text{ min}$$

b) Given the annual demand, the hourly production rate is
$$R_p = \frac{D_a}{50SH} = \frac{100,000}{50 \times 5 \times 7.5} = 53.33 \text{ units/Hr}$$

c) The corresponding cycle time $T_c$, with an uptime efficiency of 96% is
$$T_c = \frac{60E}{R_p} = \frac{60 \times 0.96}{53.33} = 1.08 \text{ min}$$

d) The theoretical minimum number of workers is given by
$$w^* = \text{Minimum Integer} \geq \frac{T_{wc}}{T_c} = \frac{4.0}{1.08} = 4 \text{ workers}$$

e) The available service time against which the line must be balanced is
$$T_s = T_c - T_r = 1.08 - 0.08 = 1.00 \text{ min.}$$

### TABLE: work elements arranged according to $T_{ek}$

<table>
<thead>
<tr>
<th>Work Element</th>
<th>$T_{ek}$ (min)</th>
<th>Preceded By</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>3,4</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
<td>9,10</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>0.38</td>
<td>5,8</td>
</tr>
<tr>
<td>7</td>
<td>0.32</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>0.27</td>
<td>6,7,8</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>0.12</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>0.11</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>1,2</td>
</tr>
</tbody>
</table>

Work elements are arranged in descending order in the above Table, and the algorithm is carried out.

### TABLE: Work Elements Assigned to Stations According to the Largest Candidate Rule

<table>
<thead>
<tr>
<th>Station</th>
<th>Work element</th>
<th>$T_{ek}$ (min)</th>
<th>Station time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.7</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>0.6</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>0.32</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>0.5</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

Five workers and stations are required in the solution. Balance efficiency is computed as:
$$E_b = \frac{T_{wc}}{wT_r} = \frac{4.0}{5 \times 1.0} = 0.80$$

Example-5:
The table below defines the precedence relationships and element times for a new model toy. (a) Construct the precedence diagram for this job. (b) If the ideal cycle time = 1.1 min, repositioning time = 0.1 min, and uptime proportion is assumed to be 1.0, what is the theoretical minimum number of workstations required to minimize the balance delay under the assumption that there will be one worker per station? (c) Use the largest candidate rule to assign work elements to stations. (d) Compute the balance delay for your solution.
### Given:

- $T_c = 1.1$ min
- $T_r = 0.1$ min
- $E = 1.0$
- $W^* = ?$
- $D = ?$

### Solution:

b) First we have to calculate the service time, i.e., $T_s = T_c - T_r = 1.1 - 0.1 = 1.0$ min

d) The theoretical minimum number of workers is given by

$$w^* = \text{Minimum Integer} \geq \frac{T_{wc}}{T_r} = \frac{4.0}{1.1} = 3.6363 \approx 4 \text{ workers}$$

d) balance delay is given by

$$d = \frac{(wT_s - T_{wc})}{wT_s} = \frac{(5 \times 1.0 - 4.3)}{5 \times 1.0} = 0.14$$

a) The precedence diagram for the given problem is shown below:

![Precedence Diagram](image)

### Work Elements Assigned to Stations According to the Largest Candidate Rule

<table>
<thead>
<tr>
<th>Work element</th>
<th>$T_{ek}$ (min)</th>
<th>Preceded by</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td>6,9</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>3,5</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>4,5</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0.3</td>
<td>7,8</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>2</td>
</tr>
</tbody>
</table>

### Work Elements Assigned to Stations According to the Largest Candidate Rule

<table>
<thead>
<tr>
<th>Station</th>
<th>Work element</th>
<th>$T_{ek}$ (min)</th>
<th>Station</th>
<th>time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.8</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0.6</td>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>0.6</td>
<td>4</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>0.5</td>
<td>5</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>0.4</td>
<td>3</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>0.3</td>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.1</td>
<td>9</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.6</td>
<td>5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Five workers and stations are required in the solution. Balance efficiency is computed as:

$$E_b = \frac{T_{wc}}{wT_s} = \frac{4.3}{5 \times 1.0} = 0.86$$
3.12.2 Kilbridge and Wester method

This method has received considerable attention since its introduction in 1961 and has been applied with apparent success to several complicated line balancing problems in industry. It is a heuristic procedure that selects work elements for assignment to stations according to their position in the precedence diagram. This overcomes one of the difficulties with the largest candidate rule in which an element may be selected because of a high $T_{ek}$ value but irrespective of its position in the precedence diagram. In general, the Kilbridge and Wester method provides a superior line balance solution than the largest candidate rule.

In the Kilbridge and Wester method,

1) Work elements in the precedence diagram are arranged into columns.
2) The elements can then be organized into a list according to their columns, with the elements in the first column listed first. If a given element can be located in more than one column, then list all of the columns for that element.
3) Apply the largest candidate rule within each column. This is helpful when assigning elements to stations; because it ensures that the larger elements are selected first, thus increasing our chances of making the sum of $T_{ek}$ in each station closer to the allowable $T_s$ limit.
4) Once the list is established, use the last three steps of largest candidate rule to assign work elements to the stations.

Example 6:

Apply kilbridge and wester method to example 4 above. Determine the balance efficiency for your solution.

Solution:

For the example 4, as we have discussed already the precedence diagram is as shown below.

Table: Work Elements Listed According to Columns from the above Figure for the Kilbridge and Wester Method

<table>
<thead>
<tr>
<th>Work Element</th>
<th>Column</th>
<th>$T_{ek}$ (min)</th>
<th>Preceded By</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>I</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>II</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>II, III</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>II</td>
<td>0.1</td>
<td>1, 2</td>
</tr>
<tr>
<td>8</td>
<td>III</td>
<td>0.6</td>
<td>3, 4</td>
</tr>
<tr>
<td>7</td>
<td>III</td>
<td>0.32</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>III</td>
<td>0.11</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>IV</td>
<td>0.38</td>
<td>5, 8</td>
</tr>
<tr>
<td>9</td>
<td>IV</td>
<td>0.27</td>
<td>6, 7, 8</td>
</tr>
<tr>
<td>11</td>
<td>V</td>
<td>0.5</td>
<td>9, 10</td>
</tr>
<tr>
<td>12</td>
<td>VI</td>
<td>0.12</td>
<td>11</td>
</tr>
</tbody>
</table>

Figure: Work elements are arranged into columns for the kilbridge and wester method

TABLE: Work Elements Assigned to Stations According to the Kilbridge and Wester Method

<table>
<thead>
<tr>
<th>Station</th>
<th>Work Element</th>
<th>Column</th>
<th>$T_{ek}$ (min)</th>
<th>Station Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>I</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>I</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>II</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>II</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>II</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>III</td>
<td>0.11</td>
<td>0.81</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>III</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>III</td>
<td>0.32</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>IV</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>IV</td>
<td>0.27</td>
<td>0.68</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>V</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>VI</td>
<td>0.12</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Work elements are arranged in order of columns in Table above. Five workers are required, and the balance efficiency is $E_b = 0.80$. Note that although the balance efficiency is the same as in the largest candidate rule, the allocation of work elements to stations is different.
Example 6:
Apply kilbridge and wester method to example 5 above. Determine the balance efficiency for your solution.

Solution:
Step 1: construct the precedence diagram. Arrange them in columns as follows.
Step 2: The elements can then be organized into a list according to their columns, with the elements in the first column listed first.
Figure: Work elements are arranged into columns for the kilbridge and wester method

<table>
<thead>
<tr>
<th>Work elements</th>
<th>Column</th>
<th>$T_{ek}$ (min)</th>
<th>Preceded by</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 I</td>
<td>I</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>3 II</td>
<td>II</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>2 II</td>
<td>II</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>6 III</td>
<td>III</td>
<td>0.6</td>
<td>3</td>
</tr>
<tr>
<td>4 III</td>
<td>III</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>5 III</td>
<td>III</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>8 IV</td>
<td>IV</td>
<td>0.5</td>
<td>3,5</td>
</tr>
<tr>
<td>7 IV</td>
<td>IV</td>
<td>0.4</td>
<td>4,5</td>
</tr>
<tr>
<td>9 V</td>
<td>V</td>
<td>0.3</td>
<td>7,8</td>
</tr>
<tr>
<td>10 VI</td>
<td>VI</td>
<td>0.6</td>
<td>6,9</td>
</tr>
</tbody>
</table>

Table: Work Elements Listed According to Columns for the Kilbridge and Wester Method

Step 4: Use the last three steps of largest candidate rule to assign work elements to the stations.

<table>
<thead>
<tr>
<th>station</th>
<th>Work element</th>
<th>Column</th>
<th>$T_{ek}$ (min)</th>
<th>Station time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 I</td>
<td>I</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 II</td>
<td>II</td>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>3 II</td>
<td>II</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4 III</td>
<td>III</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5 III</td>
<td>III</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>6</td>
<td>6 III</td>
<td>III</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7 IV</td>
<td>IV</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td>9</td>
<td>8 IV</td>
<td>IV</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>9 V</td>
<td>V</td>
<td>0.3</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>10 VI</td>
<td>VI</td>
<td>0.6</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Balance efficiency for this case is calculated as follows.

$$E_b = \frac{T_{wc}}{wT_s} = \frac{4.3}{5 \times 1.0} = 0.86$$

3.12.3 Ranked Positional Weights Method

The ranked positional weights method was introduced by Helgeson and Birnie. In this method, a ranked positional weight value (call it RPW for short) is computed for each element. The RPW takes into account both the $T_{ek}$ value and its position in the precedence diagram. Specifically, $RPW_k$ is calculated by summing $T_{ek}$ and all other times for elements that follow $T_{ek}$ in the arrow chain of the precedence diagram. Elements are compiled into a list according to their RPW value, and the algorithm proceeds using the same largest candidate rule steps.

Example 6:
Apply the ranked positional weights method to Example 4.

Solution: The RPW must be calculated for each element as follows.

<table>
<thead>
<tr>
<th>Work element</th>
<th>RPW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.30</td>
</tr>
<tr>
<td>2</td>
<td>2.67</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>1.97</td>
</tr>
<tr>
<td>5</td>
<td>1.30</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>1.21</td>
</tr>
<tr>
<td>8</td>
<td>1.87</td>
</tr>
<tr>
<td>9</td>
<td>0.89</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
</tr>
<tr>
<td>11</td>
<td>0.62</td>
</tr>
<tr>
<td>12</td>
<td>0.12</td>
</tr>
</tbody>
</table>
TABLE: List of Elements Ranked According to Their Ranked Positional Weights (RPW)

<table>
<thead>
<tr>
<th>Work Element</th>
<th>RPW</th>
<th>( T_w ) (min)</th>
<th>Preceded By</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.30</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>3.00</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2.87</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>1.97</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>8</td>
<td>1.87</td>
<td>0.6</td>
<td>3.4</td>
</tr>
<tr>
<td>5</td>
<td>1.30</td>
<td>0.3</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>1.21</td>
<td>0.32</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>0.11</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>0.36</td>
<td>5.8</td>
</tr>
<tr>
<td>9</td>
<td>0.89</td>
<td>0.27</td>
<td>6.78</td>
</tr>
<tr>
<td>11</td>
<td>0.82</td>
<td>0.9</td>
<td>9.10</td>
</tr>
<tr>
<td>12</td>
<td>0.12</td>
<td>0.12</td>
<td>11</td>
</tr>
</tbody>
</table>

Work elements are listed according to RPW value in the above Table. Note that the largest \( T_s \) value is 0.92 min. This can be exploited by operating the line at this faster rate; with the result that line balance efficiency is improved and production rate is increased.

\[
E_p = \frac{T_{we}}{wT_s} = \frac{4.0}{5 \times 0.92} = 0.87
\]

The cycle time is \( T_c = T_s + T_r = 0.92 + 0.08 = 1.00 \) Therefore,

\[
R_c = \frac{60}{T_c} = 60/1.0 = 60 \text{ cycles/hr}
\]

\[E = \frac{R_p}{R_c} = \frac{E \times X \times R_c}{0.96 \times 60} = 57.6 \text{ units/Hr}\]

This is a better solution than the previous line balancing methods provided. It turns out that the performance of a given line balancing algorithm depends on the problem to be solved. Some line balancing methods work better on some problems, while other methods work better on other problems.

### 3.13 Computerized Line Balancing Techniques

The three methods described above have all been developed into computer programs to solve large line balancing problems in industry. However, even as rapidly executed computer programs, these algorithms are still heuristic and do not guarantee an optimum solution. Attempts have been made to exploit the high speed of the digital computer by developing algorithms that either (1) perform a more exhaustive search of the set of solutions to a given problem or (2) utilize some mathematical optimization technique to solve it. Examples of the first type are COMSOL and CALB.

COMSOL (stands for COmputer Method of Sequencing Operations for Assembly Lines) was developed at Chrysler Corp. and reported by Arcus. The basic procedure is to iterate through a large number of alternative solutions by randomly assigning elements to stations in each solution, and on each iteration comparing the current solution with the previous best solution, discarding the current one if it is not better than the previous best, and replacing the previous best if the current solution is better.

During the 1970s, the Advanced Manufacturing Methods Program of the IIT Research Institute was a nucleus for research on line balancing in the United States. Around 1968, the group introduced CALB (Computer-Aided Line Balancing), which has been widely adopted in a variety of industries that manufacture assembled products, including automotive, appliances, electronic equipment, and military hardware. CALB can be used for both single model and mixed model lines.

### 3.14 Analysis of Transfer Lines with No Internal Storage

In the analysis of automated production lines, two general problem areas must be addressed:

- **Process technology**
- **Systems technology**

**Process technology** refers to a body of knowledge about the theory and principles of the particular manufacturing processes used on the production line. For example, in the machining process, process technology includes the metallurgy and machinability of the work material, the proper application of cutting tools, chip control, machining economics, machine tool vibrations, and a host of other problem areas and issues. Many of the problems encountered in machining can be solved by direct application of good machining principles. The same is true of other processes. In each process, a technology has been developed over many years of research and practice. By making use of this technology, each individual workstation in the production line can be designed to operate at or near its maximum performance. However, even if each station performance could be optimized, there still remain larger systems issues that must be analyzed.
It is with this viewpoint of the larger system that we identify the second problem area. Two aspects of this problem stand out. The first is the **line balancing problem**. Although this problem is normally associated with the design of manual assembly lines, it is also a problem on automated production lines. Somehow, the total machining work that must be accomplished on the automated line must be divided as evenly as possible among the workstations. The solution to this problem on a machining transfer line is usually dominated by technological considerations. Certain machining operations must be performed before others (e.g., drilling must precede tapping), and the element times are determined by the cycle time required to accomplish the given machining operation at a station. These two factors make the line balancing problem less of an issue on a machining type production line than it is in manual assembly, where the total work content can be divided into much smaller work elements, and the possible permutations on the order in which the elements can be performed is much greater.

The second and more critical systems problem in automated production line design is the **reliability problem**. In a highly complex and integrated system such as an automated production line, failure of any one component can stop the entire system. It is this problem of how line performance is affected by reliability that we consider in this section.

**Common Reasons for Downtime on an Automated Production Line**

- Tool failures at workstations
- Tool adjustments at workstations
- Scheduled tool changes
- Limit switch or other electrical malfunctions
- Mechanical failure of a workstation
- Mechanical failure of the transfer system
- Stockouts of starting work units
- Insufficient space for completed parts
- Preventive maintenance on the line
- Worker breaks

### 3.14.1 Basic Terminology and Performance Measures

We make the following assumptions about the operation of the transfer lines and rotary indexing machines:

- The workstations perform processing operations, such as machining, not assembly
- Processing times at each station are constant, though not necessarily equal
- Synchronous transfer of parts
- No internal storage buffers

In the operation of an automated production line, parts are introduced into the first workstation and are processed and transported at regular intervals to succeeding stations. This interval defines the ideal cycle time $T_c$ of the production line. $T_c$ is the processing time for the slowest station on the line plus the transfer time; that is,

$$T_c = \text{Max} \{T_s\} + T_r$$

Where $T_c$ = ideal cycle time on the line (min); $T_s$ = the processing time at station i (min); and $T_r$ = repositioning time, called the transfer time here (min). We use the Max $\{T_s\}$ because this longest service time establishes the pace of the production line. The remaining stations with smaller service times must wait for the slowest station. Therefore, these other stations will experience idle time. The situation is the same as for a manual assembly line.

In the operation of a transfer line, random breakdowns and planned stoppages cause downtime on the line. Although the breakdowns and line stoppages occur randomly, their frequency can be measured over the long run. When the line stops, it is down a certain average time for each downtime occurrence. These downtime occurrences cause the actual average production cycle time of the line to be longer than the ideal cycle time. We can formulate the following expression for the actual average production time $T_p$:

$$T_p = T_c + FT_d$$

Where $F = \text{downtime frequency, line stops/cycle}$; and $T_d = \text{downtime per line stop, min}$. The downtime $T_d$ includes the time for the repair crew to swing into action, diagnose the cause of the failure, fix it, and restart the line. Thus, $FT_d = \text{downtime averaged on a per cycle basis}$.

One of the important measures of performance on an automated transfer line is production rate, which can be computed as the reciprocal of $T_p$:

$$R_p = \frac{1}{T_p}$$

Where $R_p = \text{actual average production rate (pc/min)}$, and $T_p$ is the actual average production time (min). It is of interest to compare this rate with the ideal production rate given by

$$R_i = \frac{1}{T_c}$$

Where $R_i = \text{ideal production rate (pc/min)}$. It is customary to express production rates on automated production lines as hourly rates.
In the context of automated production systems, line efficiency refers to the proportion of uptime on the line and is really a measure of reliability more than efficiency. Nevertheless, this is the terminology of production lines. Line efficiency can be calculated as follows:

\[ E = \frac{T_c}{T_p} = \frac{T_c}{T_c + FT_d} \]

Where \( E \) = the proportion of uptime on the production line.

An alternative measure of performance is the proportion of downtime on the line, which is given by

\[ D = \frac{FT_d}{T_p} = \frac{FT_d}{T_c + FT_d} \]

Where \( D \) = proportion of downtime on the line. It is obvious that \( E + D = 1.0 \).

An important economic measure of performance of an automated production line is the cost per unit produced. This piece cost includes the cost of the starting blank that is to be processed on the line, the cost of time on the production line, and the cost of any tooling that is consumed (e.g., cutting tools on a machining line). The piece cost can be expressed as the sum of these three factors:

\[ C_{pc} = C_m + C_o T_p + C_t \]

Where \( C_{pc} \) = cost per piece ($/pc); \( C_m \) = cost of starting material ($/pc); \( C_o \) = cost per minute to operate the line ($/min); \( T_p \) = average production time per piece (min/pc); and \( C_t \) = cost of tooling per piece ($/pc). \( C_o \) includes the allocation of the capital cost of the equipment over its expected service life, labor to operate the line, applicable overheads, maintenance, and other relevant costs, all reduced to a cost per minute.

Example 7:
A 20-station transfer line is being proposed to machine a certain component currently produced by conventional methods. The proposal received from the machine tool builder states that the line will operate at a production rate of 50 pc/hr at 100% efficiency. From similar transfer lines, it is estimated that breakdowns of all types will occur with a frequency \( F = 0.10 \) breakdown per cycle and that the average downtime per line stop will be 8.0 min. The starting casting that is machined on the line costs $3.00 per part. The line operates at a cost of $75.00/hr. The 20 cutting tools (one tool per station) last for 50 parts each, and the average cost per tool = $2.00 per cutting edge. Based on this data, compute the following: (a) production rate, (b) line efficiency, and (c) cost-per unit piece produced on the line.

Given:
\[ n = 20 \text{ stations} \]
\[ R_c = 50 \text{ Pc/Hr} \]
\[ (\text{As } E= 100 \%) \]
\[ F = 0.10 \]
\[ T_d = 8.0 \text{ min} \]
\[ C_m = $3.00 \text{ per part} \]
\[ C_o = $75.00/\text{hr} \]
\[ C_t = $2.00 \text{ per cutting edge} \]
\[ n_t = 20 \text{ for 50 parts} \]

Solution:
(a) At 100% efficiency, the line produces 50 pc/hr. The reciprocal gives the ideal cycle time per piece:
\[ \frac{1}{50} = 0.02 \text{ hr/pc} = 0.02 \times 60 \text{ min/pc} = 1.2 \text{ min/pc} \]

The average production time per piece is given by
\[ T_p = T_c + FT_d = 1.2 + 0.10(8.0) = 1.2 + 0.8 = 2.0 \text{ min/pc} \]

Production rate is the reciprocal of production time per piece:
\[ R_p = \frac{1}{T_p} = 0.50 \text{ pc/min} = 0.5 \times 60 \text{ pc/hr} = 30.0 \text{ pc/hr} \]

Efficiency is the ratio of ideal cycle time to actual average production time,
\[ E = \frac{T_c}{T_p} = \frac{1.2}{2.0} = 0.60 = 60\% \]

Finally, for the cost per piece produced, we need the tooling cost per piece, which is computed as follows:
\[ C_{pc} = (20 \text{ tools}) \times ($2/\text{tool}) / (50 \text{ parts}) = $0.80/\text{pc} \]

Now the unit cost can be calculated by Eq:
\[ C_{pc} = C_m + C_o T_p + C_t \]

The hourly rate of $75/hr to operate the line is equivalent to $1.25/min.

( because \( C_o \) should be cost to operate the line per min ; 75/60 = 1.25)
\[ C_{pc} = 3.00 + 1.25(2.0) + 0.80 = $6.30/\text{pc} \]

Example 8:
A ten-station transfer machine has an ideal cycle time of 30 sec. The frequency of line stops is \( F = 0.075 \) stops per cycle. When line stops occurs, the average downtime is 4.0 min. Determine (a) average production rate in pc/hr, (b) line efficiency, and (c) proportion downtime. If the Cost elements associated with the operation of the ten-station transfer line are; raw work part cost = $0.55/pc, line operating cost = $42.00/hr, and cost of disposable tooling = $0.27/pc. (d) Compute the average cost of a work piece produced.

Given:
\[ n = 10 \text{ stations} \]
\[ T_c = 30 \text{ sec} = 0.5 \text{ min} \]

Solution:
(a) The average production time per piece is given by
\[ T_p = T_c + FT_d = 0.5 + 0.075(4.0) = 0.8 \text{ min/pc} \]
Let $p_i$ be the probability that the given part will jam at any station $i$. Summing all these probabilities from $i = 1$ through $i = n$ gives the probability or frequency of line stops per cycle. Fortunately there is an easier way to

$$F = \sum_{i=1}^{n} p_i$$

Where $F$ = expected frequency of line stops per cycle, $p_i$ = frequency of station breakdown per cycle, causing a line stop; and $n$ = number of workstations on the line. If all $p_i$ are assumed equal, which is unlikely but useful for approximation and computation purposes, then

$$F = nP$$

Where all $p_i$ are equal, $p_1 = p_2 = ... = p_n = p$.

### Lower-Bound Approach:

The lower-bound approach gives an estimate of the lower limit on the expected frequency of line stops per cycle. In this approach, we assume that a station breakdown results in destruction of the part, resulting in its removal from the line and preventing its subsequent processing at the remaining workstations.

Again let $p_i$ = the probability that a workpart will jam at a particular station $i$. Then, considering a given part as it proceeds through the line, $p_1$ = probability that the part will jam at station 1, and $(1 – p_1)$ = probability that the part will not jam at station 1 and will thus be available for processing at subsequent stations. A jam at station 2 is dependent on successfully making it through station 1 and therefore the probability that, this same part will jam at station 2 is given by $p_2(1 – p_1)$. Generalizing, the quantity

$$P_i(1-p_{i-1}) (1-p_{i-2})....(1-P_2)(1-P_1)$$

where $i = 1, 2, ..., n$

is the probability that the given part will jam at any station $i$. Summing all these probabilities from $i = 1$ through $i = n$ gives the probability or frequency of line stops per cycle. Fortunately there is an easier way to
determine this frequency by taking note of the fact that the probability that a given part will pass through all n stations without a line stop is

\[ F = 1 - \prod_{i=1}^{n} (1 - p_i) \]

Therefore, the frequency of line stops per cycle is

\[ F = 1 - \prod_{i=1}^{n} (1 - p_i) \]

If all probabilities \( p_i \) are equal, \( p_i = p \), then

\[ F = 1 - (1 - p)^n \]

Because of parts removal in the lower-bound approach, the number of parts coming off the line is less than the number launched onto the front of the line. Therefore, the production rate formula must be amended to reflect this reduction in output. Given that \( F \) = frequency of line stops and a part is removed for every line stop, then the proportion of parts removed from the line is \( F \). Accordingly, the proportion of parts produced is \( (1 - F) \). This is the yield of the production line. The production rate equation becomes the following:

\[ R_{ap} = \frac{1 - F}{T_p} \]

Where \( R_{ap} \) = the average actual production rate of acceptable parts from the line; \( T_p \) = the average cycle rate of the transfer machine. \( R_p \), which is the reciprocal of \( T_p \), is the average cycle rate of the system.

Example 9:
A 20-station transfer line has an ideal cycle time \( T_c = 1.2 \) min. The probability of station breakdowns per cycle is equal for all stations, and \( p = 0.005 \) breakdowns/cycle and the average downtime per line stop will be 8.0 min. For each of the upper-bound and lower-bound approaches, determine (a) frequency of line stops per cycle, (b) average actual production rate, and (c) line efficiency.

Given:
- \( n = 20 \) stations
- \( T_c = 1.2 \) min
- \( P = 0.005 \) breakdown/cycle
- \( F = ? \)
- \( R_{ap} = ? \)
- \( E = ? \)

i) For upper-Bound approach
a) frequency of line stops cycle
\[ F = nP = 20 \times 0.005 = 0.10 \] line stops/cycle

b) The average production time per piece is given by
\[ T_p = T_c + FT_d = 1.2 + 0.10(8.0) = 1.2 + 0.8 = 2.0 \] min/pc.

Production rate is the reciprocal of production time per piece:
\[ R_p = \frac{1}{T_p} = \frac{1}{2.0} = 0.500 \] pc/min = 0.5X(60) pc/hr = 30.0 pc/hr

(60 min is multiplied with 0.5 to convert it to hr)

c) Efficiency is the ratio of ideal cycle time to actual average production time,
\[ E = \frac{T_c}{T_p} = \frac{1.2}{2.0} = 0.60 = 60\% \]

ii) For Lower-Bound approach
a) frequency of line stops cycle
\[ F = (1 - (1 - p)^n) = (1 - (1 - 0.005)^{20}) = 0.0954 \] line stops per cycle

( where \( n \) = number of stations)

b) The average production time per piece is given by
(For the lower-bound approach, we must calculate \( T_p \) using the new value of \( F \))
\[ T_p = T_c + FT_d = 1.2 + 0.0954(8.0) = 1.9631 \] min

Now compute production rate,
we have \[ R_{ap} = \frac{1 - F}{T_p} = \frac{(1 - 0.0954)}{1.9631} = 0.4608 \] Pc/min
\[ = 0.4608 \times 60 = 27.65 \] PC/hr

c) Efficiency is the ratio of ideal cycle time to actual average production time,
\[ E = \frac{T_c}{T_p} = \frac{1.2}{1.9631} = 0.6113 = 61.13\% \]

Example 10:
A 22-station in-line transfer machine has an ideal cycle time of 0.35 min. Station breakdowns occur with a probability \( p = 0.01 \). Assume that station breakdowns are the only reason for line stops. Average downtime = 8.0 min per line stop. Use the upper-bound approach to determine: (a) ideal production rate \( R_c \) (b) frequency of line stops \( F \), (c) average actual production rate \( R_{ap} \), and (d) line efficiency \( E \).
Given: n = 22 stations
Tc = 0.35 min
Td = 8.0 min
P = 0.01
Rc = ?
F = ?
Rp = ?
E = ?

Solution: For the upper-bound approach

a) We know that, the ideal production rate is the reciprocal of ideal cycle time

\[ R_c = \frac{1}{T_c} = \frac{1}{0.35} = 2.8571 \text{ Pc/min} = 2.8571 \times 60 = 171.42 \text{ Pc/hr} \]

b) frequency of line stops F is given by

\[ F = nP = 22 \times 0.01 = 0.22 \text{ line stops/cycle} \]

c) average actual production rate \( R_p \) is given by,

\[ R_p = \text{wkt, the average production rate is the reciprocal of actual production time} \]

\[ T_p = T_c + FT_d = 0.35 + 0.22(8.0) = 2.11 \text{ min/pc.} \]

Production rate is the reciprocal of production time per piece:

\[ R_p = \frac{1}{T_p} = \frac{1}{2.11} = 0.474 \text{ pc/min} = 0.474 \times (60) \text{ pc/hr} = 28.43 \text{ pc/hr} \]

(60 min is multiplied with 0.5 to convert it to hr)

d) Efficiency is the ratio of ideal cycle time to actual average production time,

\[ E = \frac{T_c}{T_p} = \frac{0.35}{2.11} = 0.1658 = 16.58\% \]

3.15 Analysis of Transfer Lines with Storage Buffers

In an automated production line with no internal parts storage, the workstations are interdependent. When one station breaks down, all other stations on the line are affected, either immediately or by the end of a few cycles of operation. The other stations will be forced to stop for one of two reasons:

- Starving of stations
- Blocking of stations

These terms have meanings that are basically the same as in the operation of manual assembly lines. Starving on an automated production line means that a workstation is prevented from performing its cycle because it has no part to work on. When a breakdown occurs at any workstation on the line, the stations downstream from (following) the affected station will either immediately or eventually become starved for parts.

Blocking means that a station is prevented from performing its work cycle because it cannot pass the part it just completed to the neighboring downstream station. When a breakdown occurs at a station on the line, the stations upstream from (preceding) the affected station become blocked because the broken down station cannot accept the next part for processing from the neighboring upstream station. Therefore, none of the upstream stations can pass their just completed parts forward.

A method by which production lines can be designed to operate more efficiently is to add one or more parts storage buffers between workstations. The storage buffer divides the line into stages that can operate independently for a number of cycles, the number depending on the storage capacity of the buffer. If one storage buffer is used, the line is divided into two stages. If two buffers are used at two different locations along the line, then a three-stage line is formed. And so forth. The upper limit on the number of storage buffers is to have storage between every pair of adjacent stations. The number of stages will then equal the number of workstations. For an \( n \)-stage line, there will be \( n-1 \) storage buffers. This, of course, does not include the raw parts inventory at the front of the line or the finished parts inventory that accumulates at the end of the line.

Consider a two-stage transfer line, with a storage buffer separating the stages. Let us suppose that, on average, the storage buffer is half full. If the first stage breaks down, the second stage can continue to operate (avoid starving) using parts that have been collected in the buffer. And if the second stage breaks down, the first stage can continue to operate (avoid blocking) because it has the buffer to receive its output. The reasoning for a two-stage line can be extended to production lines with more than two stages. For any number of stages in an automated production line, the storage buffers allow each stage to operate somewhat independently, the degree of independence depending on the capacity of the upstream and downstream buffers.

3.15.1 Limits of Storage Buffer Effectiveness

Two extreme cases of storage buffer effectiveness can be identified:

- No buffer storage capacity at all and
- Infinite capacity storage buffers.

In the analysis that follows, let us assume that the ideal cycle time \( T_c \) is the same for all stages considered. This is generally desirable in practice because it helps to balance production rates among stages.

In the case of no storage capacity, the production line acts as one stage. When a station breaks down, the entire line stops. This is the case of a production line with no internal storage analyzed in Section. The efficiency of the line was given by Eq.

\[ E = \frac{T_c}{T_p} = \frac{T_c}{T_c + FT_d} \]
We rewrite it here as the line efficiency of a zero capacity storage buffer:

$$E_0 = \frac{T_c}{T_c + FT_d}$$

where the subscript 0 identifies $E_0$ as the efficiency of a line with zero storage buffer capacity; and the other terms have the same meanings as before.

The opposite extreme is the case where buffer zones of infinite capacity are installed between every pair of stages. If we assume that each buffer zone is half full (in other words, each buffer zone has an infinite supply of parts as well as the capacity to accept an infinite number of additional parts), then each stage is independent of the rest. The presence of infinite storage buffers means that no stage will ever be blocked or starved because of a breakdown at some other stage.

Of course, an infinite capacity storage buffer cannot be realized in practice. If it could, then the overall line efficiency would be limited by the bottleneck stage. That is, production on all other stages would ultimately be restricted by the slowest stage. The downstream stages could only process parts at the output rate of the bottleneck stage. And it would make no sense to run the upstream stages at higher production rates because this would only accumulate inventory in the storage buffer ahead of the bottleneck. As a practical matter, therefore, the upper limit on the efficiency of the entire line is defined by the efficiency of the bottleneck stage. Given that the cycle time $T_c$ is the same for all stages, the efficiency of any stage $k$ is given by:

$$E_k = \frac{T_c}{T_c + F_k T_{dk}}$$

Where the subscript $k$ is used to identify the stage. According to our argument above, the overall line efficiency would be given by

$$E_\infty = \text{Minimum}\{E_k\}$$

Where the subscript $\infty$ identifies $E_\infty$ as the efficiency of a line whose storage buffers all have infinite capacity.

By including one or more storage buffers in an automated production line, we expect to improve the line efficiency above $E_0$, but we cannot expect to achieve $E_\infty$ simply because buffer zones of infinite capacity are not possible. Hence, the actual value of line efficiency for a given buffer capacity $b$ will fall somewhere between these extremes:

$$E_0 < E_b < E_\infty$$

### 3.15.2 Analysis of a Two-Stage Transfer Line

The two-stage line is divided by a storage buffer of capacity $b$, expressed in terms of the number of work parts that it can store. The buffer receives the output of stage 1 and forwards it to stage 2, temporarily storing any parts not immediately needed by stage 2 up to its capacity $b$. The ideal cycle time $T_c$ is the same for both stages. We assume the downtime distributions of each stage to be the same with mean downtime $= T_d$. Let $F_1$ and $F_2$ = the breakdown rates of stages 1 and 2, respectively. $F_1$ and $F_2$ are not necessarily equal.

Over the long run, both stages must have equal efficiencies. If the efficiency of stage 1 were greater than the stage 2 efficiency, then inventory would build up in the storage buffer until its capacity $b$ is reached. Thereafter, stage 1 would eventually be blocked, when it out produced stage 2. Similarly, if the efficiency of stage 2 were greater than that of stage 1, the inventory in the buffer would become depleted, thus starving stage 2. Accordingly, the efficiencies in the two stages would tend to equalize over time. The overall line efficiency for the two-stage line can be expressed:

$$E_b = E_0 + D_1 h(b) E_2$$

where $E_b$ = overall line efficiency for a two-stage line with buffer capacity $b$; $E_0$ = line efficiency for the same line with no internal storage; and the second term on the right-hand side ($D_1 h(b) E_2$) represents the improvement in efficiency that results from having a storage buffer with $b > 0$. Let us examine the right-hand side terms in the above Eq.

The value of $E_0$ was given by Eq.

$$E_0 = \frac{T_c}{T_c + FT_d}$$

But we write it below to explicitly define the two-stage efficiency when $b = 0$:

$$E_0 = \frac{T_c}{T_c + (F_1 + F_2)T_d}$$

The term $D_1$ can be thought of as the proportion of total time that stage 1 is down, defined as follows:

$$D_1 = \frac{F_1 T_d}{T_c + (F_1 + F_2)T_d}$$
The term \( h(b) \) is the proportion of the downtime \( D_1 \) (when stage 1 is down) that stage 2 could be up and operating within the limits of storage buffer capacity \( b \).

Finally, \( E_2 \) corrects for the assumption in the calculation of \( h(b) \) that both stages are never down at the same time. This assumption is unrealistic. What is more-realistic is that when stage 1 is down but stage 2 could be producing because of parts stored in the buffer; there will be times when stage 2 itself breaks down. Therefore, \( E_2 \) provides an estimate of the proportion of stage 2 uptime when it could otherwise be operating even with stage 1 being down. \( E_2 \) is calculated as:

\[
E_2 = \frac{T_c}{T_c + F_c T_d}
\]

The average production rate can be evaluated based on the knowledge of ideal cycle time and line efficiency. According to the equation \( E = \frac{T_c}{T_p} \), since \( R_p \) is the reciprocal of \( T_p \), then \( E = T_c R_p \). Rearranging, we have,

\[
R_p = \frac{E}{T_c}
\]

The value of \( h(b) \) can be calculated as follows for different cases as mentioned below.

**Assumptions and definitions:** Assume that the two stages have equal downtime distributions \( (T_{d1} = T_{d2} = T_d) \) and equal cycle times \( (T_{c1} = T_{c2} = T_c) \). Let \( F_1 \) = downtime frequency for stage 1, and \( F_2 \) = downtime frequency for stage 2. Define \( r \) to be the ratio of breakdown frequencies as follows:

**For Constant downtime situation:** Each downtime occurrence is assumed to be of constant duration \( T_d \). This is a case of no downtime variation. Given buffer capacity \( b \), define \( B \) and \( L \) as follows:

\[
b = B \frac{T_d}{T_c} + L
\]

Where \( B \) is the largest integer satisfying the relation: \( B \frac{T_d}{T_c} \geq B \), and \( L \) represents the leftover units, the amount by which \( b \) exceeds \( B \frac{T_d}{T_c} \). There are two cases:

Case 1: \( r = 1.0 \); \( h(b) = \frac{B}{B + 1} + \frac{L}{T_d} \frac{T_c}{T_d} \frac{1}{(B + 1)(B + 2)} \)

Case 2: \( r \neq 1.0 \); \( h(b) = \frac{1 - r^B}{1 - r^{B+1}} + \frac{L}{T_d} \frac{T_c}{T_d} \frac{r^{B+1}(1 - r)^2}{(1 - r^{B+1})(1 - r^{B+2})} \)

**For Geometric downtime distribution:** In this downtime distribution, the probability that repairs are completed during any cycle duration \( T_c \) is independent of the time since repairs began. This is a case of maximum downtime variation. There are two cases:

Case 1: \( r = 1.0 \); \( h(b) = \frac{b}{2 + (b - 1) \frac{T_c}{T_d}} \frac{T_c}{T_d} \)

Case 2: \( r \neq 1.0 \); Define \( k = \frac{T_c}{T_d} \) then \( h(b) = \frac{r(1 - k^b)}{1 - rk^b} \)

**Example 11:**
A 20-station transfer line is divided into two stages of 10 stations each. The ideal cycle time of each stage is \( T_c = 1.2 \) min. All of the stations in the line have the same probability of stopping, \( p = 0.005 \). We assume that the downtime is constant when a breakdown occurs, \( T_d = 8.0 \) min.

1) Using the upper-bound approach, compute the line efficiency for the following buffer capacities: (a) \( b = 0 \), (b) \( b = \infty \), (c) \( b = 10 \), and (d) \( b = 100 \).

2) Compute the production rate for the four cases.
3) Evaluate the efficiency, if geometric downtime distribution is assumed.
4) Compare line efficiencies and production rates for the following cases; where in each case the buffer capacity is infinite: (a) no storage buffers, (b) one buffer, (c) three buffers, and (d) 19 buffers. Assume in cases (b) and (c) that the buffers are located in the line to equalize the downtime frequencies; that is, all $F_i$ are equal.

**Given:**

1) $n = 20$ stations
   - $n_1 = 10$ stations
   - $n_2 = 10$ stations
2) $T_c = 1.2$ min
3) $P = 0.005$
4) $T_{d1} = T_{d2} = T_d = 8.0$ min
5) $E_0 = ?$
6) $E_\infty = ?$
7) $E_{10} = ?$
8) $E_{100} = ?$

**Solution:**

1) for upper bound approach

a) For a two-stage line with $20$ stations and $p = 0.005$

\[ F = np = 20(0.005) = 0.10 \]

\[ T_c = \frac{1.2}{T_c + FT_d} = \frac{1.2}{1.2 + 0.10 \times 8.0} = 0.60 \]

b) For a two-stage line with $20$ stations (each stage = $10$ stations) and $b = \infty$, we first compute $F$:

\[ F_1 = F_2 = np = 10(0.005) = 0.05 \]

\[ E_\infty = E_\infty = E_\infty = \frac{T_c}{T_c + FT_d} = \frac{1.2}{1.2 + 0.05 \times 8.0} = 0.75 \]

c) For a two-stage line with $b = 10$, we must determine each of the terms in Eq.

\[ E_{10} = E_0 + D_1 h(b) E_2 \]

In the above equation, we have already computed $E_0$ and $E_2$, next let now determine $D_1$ and $h(b)$

\[ D_1 = \frac{F_1 T_d}{T_c + (F_1 + F_2) T_d} = \frac{0.05 \times 8.0}{1.2 + (0.05 + 0.05) \times 8.0} = 0.20 \]

This is the case of equal downtime distribution, therefore, lets make use of the equation

\[ h(b) = \frac{B}{B + 1} + L \frac{T_c}{T_d} \frac{1}{(B + 1)(B + 2)} \]

since $r = F_1/F_2 = 0.05/0.05 = 1.0$

Where $b \frac{T_c}{T_d} \geq B$

\[ B = 10 \frac{1.2}{8.0} \geq 1.5 \]

As $B$ is the largest integer satisfying the above relation $B = 1$

Substitute the value of $B$ and $b$ in the equation

\[ b = B \frac{T_c}{T_d} + L ; L = b - B \frac{T_d}{T_c} = 10 - 1.2 = 3.33 \]

Therefore;

\[ h(10) = \frac{1}{1+1} + 3.33 \left(1.2\right) \frac{1}{8.0} \frac{1}{(1 + 1)(1 + 2)} = 0.5833 \]

We can now use the equation $E_{10}$ to determine the efficiency

\[ E_{10} = E_0 + D_1 h(b) E_2 = 0.60 + 0.20(0.5833)(0.75) = 0.6875 \]

d) For $b = 100$, the only the parameter in $E_b$ equation that is different from part c) is $h(b)$. lets now calculate the $h(b)$ for $b = 100$.

\[ b \frac{T_c}{T_d} \geq B \]

\[ B = 100 \frac{1.2}{8.0} \geq 15 \]

As $B$ is the largest integer satisfying the above relation $B = 15$

Substitute the value of $B$ and $b$ in the equation

\[ b = B \frac{T_c}{T_d} + L ; L = b - B \frac{T_d}{T_c} = 100 - 15 \frac{8.0}{1.2} = 0 ; i.e., L = 0 \]

Therefore;

\[ h(10) = \frac{15}{15+1} + 0 \left(1.2\right) \frac{1}{8.0} \frac{1}{(1 + 1)(1 + 2)} = 0.9375 \]

We can now use the equation $E_{100}$ to determine the efficiency

\[ E_{100} = E_0 + D_1 h(b) E_2 = 0.60 + 0.20(0.9375)(0.75) = 0.7406 \]
2) To compute the production rate for all the four cases;

a) For b = 0, $E_o = 0.60$; $R_p = \frac{E_o}{T_c} = 0.60/1.2 = 0.5 \text{ pc/min} = 30 \text{ pc/hr}$

b) For $b = \infty$, $E_\infty = 0.75$; $R_p = \frac{E_\infty}{T_c} = 0.75/1.2 = 0.625 \text{ pc/min} = 37.5 \text{ pc/hr}$

c) For $b = 10$, $E_{10} = 0.6875$; $R_p = \frac{E_{10}}{T_c} = 0.6875/1.2 = 0.5729 \text{ pc/min} = 34.375 \text{ pc/hr}$

d) For $b = 1000$, $E_{100} = 0.7406$; $R_p = \frac{E_{100}}{T_c} = 0.7406/1.2 = 0.6172 \text{ pc/min} = 37.03 \text{ pc/hr}$

3) If the geometric downtime distribution is assumed between the two stages,

a) The value of $E_o$ is already computed in case 1) and is equal to $E_o = 0.60$

b) the value of $E_\infty$ is also computed in case 1) and is equal to $E_\infty = 0.75$

c) For $b = 10$, all the parameters in $E_{10}$ equation remains same except $h(b)$. Lets find the value of $h(b)$ using equation

$$h(b) = \frac{b \frac{T_c}{T_d}}{2 + (b-1) \frac{T_c}{T_d}}$$

since geometric downtime distribution is assumed and $r = F_1/F_2 = 0.05/0.05 = 1.0$

$$h(10) = \frac{10 \frac{1.2}{8.0}}{2 + (10-1) \frac{1.2}{8.0}} = 0.4478$$

We can now use the equation $E_{10}$ to determine the efficiency

$$E_{10} = E_o + D_1 h(b) E_2 = 0.60 + 0.20(0.4478)(0.75) = 0.6875 = 0.6672$$

d) For $b = 100$, again only change is $h(b)$

$$h(100) = \frac{100 \frac{1.2}{8.0}}{2 + (100-1) \frac{1.2}{8.0}} = 0.8902$$

We can now use the equation $E_{100}$ to determine the efficiency

$$E_{100} = E_o + D_1 h(b) E_2 = 0.60 + 0.20(0.8902)(0.75) = 0.6875 = 0.6672$$

4) Efficiency and production rate for more than two buffers

a) For No storage buffers:

For no storage buffers, efficiency and production rates are already calculated in case 1) and in case 2) and are equal to $E_o = 0.60$ and $R_p = 30 \text{ pc/hr}$

b) One buffer

For one buffer (i.e., one buffer means two stage transfer line and one buffer with infinite capacity is assumed), efficiency and production rates are already calculated in case 1) and in case 2) and are equal to $E_\infty = 0.75$ and $R_p = 37.5 \text{ pc/hr}$

c) Three buffers

For three buffers (a four stage line), we have $F_1 = F_2 = F_3 = F_4 = np = 5(0.005) = 0.025$

(n = 5 since 20 stations are divided for four stages)

The production time is given by

$$T_p = T_c + FT_p = 1.2+0.025(8.0) = 1.4 \text{ min/pc}$$

We know that, the Efficiency is given by; $E_o = T_c/T_p = 1.2/1.4 = 0.8571$
The production rate is given by: \( R_p = \frac{E_\infty}{T_c} = \frac{0.8571}{1.2} = 42.86 \text{ pc/min} \)

**d) 19 buffers**

For 19 buffers (a 20 stage transfer line), we have \( F_1 = F_2 = F_3 \ldots F_{20} = np = 1(0.005) = 0.005 \)  
(n = 1 since 20 stations are divided for 20 stages)

The production rate is given by  
\[
T_p = T_c + FT_d = \frac{1.2}{1.24} \approx 0.9677
\]

We know that, the efficiency is given by: \( E_\infty = \frac{T_c}{T_p} = 1.2/1.24 = 0.9677 \)

The production rate is given by: \( R_p = \frac{E_\infty}{T_c} \)

**Example 12:**

A 30-station transfer line has an ideal cycle time \( T_c = 0.75 \text{ min} \), an average downtime \( T_d = 6.0 \text{ min} \) per line stop occurrence, and a station failure frequency \( p = 0.01 \) for all stations. A proposal has been submitted to locate a storage buffer between stations 15 and 16 to improve line efficiency. Using the upper-bound approach, determine: (a) the current line efficiency and production rate and (b) the maximum possible line efficiency and production rate that would result from installing the storage buffer.

Given:

- \( N = 30 \) stations
- \( T_c = 0.75 \text{ min} \)
- \( T_d = 6.0 \text{ min} \)
- \( p = 0.01 \)

(a) Assuming there is no buffer in the transfer line;

\[
F = np = 30(0.001) = 0.03
\]

\[
E_0 = \frac{T_c}{T_c + FT_d} = \frac{0.75}{0.75 + 0.03 \times 6.0} = 0.8064
\]

For \( b = 0 \), \( E_0 = 0.8064 \); \( R_p = \frac{E_0}{T_c} = 0.8064/0.75 = 1.0752 \text{ pc/min} = 64.51 \text{ pc/hr} \)

(b) Assuming one buffer between stations 15 and 16 (there are two stages)

\[
F = np = 15(0.001) = 0.015
\]

(n = 15, since 30 stations are divided for two stages)

\[
E_\infty = \frac{T_c}{T_c + FT_d} = \frac{0.75}{0.75 + 0.015 \times 6.0} = 0.8928
\]

For \( b = \infty \), \( E_\infty = 0.8928 \); \( R_p = \frac{E_\infty}{T_c} = 0.8928/0.75 = 1.1904 \text{ pc/min} = 71.42 \text{ pc/hr} \)